Package ‘IRTest’

September 19, 2023

Type Package

Title Parameter Estimation of Item Response Theory with Estimation of Latent Distribution

Version 1.11.0

Description Item response theory (IRT) parameter estimation using marginal maximum likelihood and expectation-maximization algorithm
(Bock & Aitkin, 1981 <doi:10.1007/BF02293801>).
Within parameter estimation algorithm, several methods for latent distribution estimation are available
(Li, 2022 <http://www.riss.kr/link?id=T16374105>).
Reflecting some features of the true latent distribution, these latent distribution estimation methods can possibly enhance the estimation accuracy and free the normality assumption on the latent distribution.

License GPL (>= 3)

Encoding UTF-8

LazyData true

RoxygenNote 7.2.3

URL https://github.com/SeewooLi/IRTest

BugReports https://github.com/SeewooLi/IRTest/issues

Suggests knitr, rmarkdown, testthat (>= 3.0.0), V8, gridExtra

VignetteBuilder knitr

Imports betafunctions, dcurver, ggplot2

Depends R (>= 2.10)

Config/testthat/edition 3

NeedsCompilation no

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R topics documented:

`cat_clps` *A recommendation for the category collapsing of items based on item parameters*

Description

In a polytomous item, one or more score categories may not have the highest probability among the categories in an acceptable θ range. In this case, the category could be regarded as a redundant one in a psychometric point of view and can be collapsed into another score category. This function returns a recommendation for a recategorization scheme based on item parameters.

Usage

```
cat_clps(item.matrix, range = c(-4, 4), increment = 0.005)
```

Arguments

`item.matrix`	A matrix of item parameters.
`range`	A range of θ to be evaluated.
`increment`	A width of the grid scheme.

Value

A list of recommended recategorization for each item.

Author(s)

Seewoo Li <cu@yonsei.ac.kr>

DataGeneration *Generating artificial item response data*

Description

This function generates artificial item response data with users specified item types, details of item parameters, and latent distribution.

Usage

```
DataGeneration(
  seed = 1,
  N = 2000,
  nitem_D = 0,
  nitem_P = 0,
  model_D = "2PL",
  model_P = "GPCM",
  latent_dist = NULL,
  item_D = NULL,
  item_P = NULL,
  theta = NULL,
  prob = 0.5,
  d = 1.7,
  sd_ratio = 1,
  m = 0,
  s = 1,
  a_l = 0.8,
  a_u = 2.5,
  b_m = NULL,
  b_sd = NULL,
  c_l = 0,
  c_u = 0.2,
  categ = NULL
)
```

Arguments

seed	A numeric value that is used on random sampling. The seed number can guarantee the replicability of the result.
N	A numeric value. The number of examinees.
nitem_D	A numeric value. The number of dichotomous items.
nitem_P	A numeric value. The number of polytomous items.

model_D	A vector of length nitem_D. The *i*th element is the probability model for the *i*th dichotomous item.
model_P	A character string that represents the probability model for the polytomous items.
latent_dist	A character string that determines the type of latent distribution. Currently available options are "beta" (four-parameter beta distribution; rBeta.4P), "chi" (χ^2 distribution; rchisq), "normal", "Normal", or "N" (standard normal distribution; rnorm), and "Mixture" or "2NM" (two-component Gaussian mixture distribution; see Li (2021) for details.)
item_D	Default is NULL. An item parameter matrix can be specified. The number of columns should be 3: a parameter for the first, b parameter for the second, and c parameter for the third column.
item_P	Default is NULL. An item parameter matrix can be specified. The number of columns should be 7: a parameter for the first, and b parameters for the rest of the columns.
theta	Default is NULL. An ability parameter vector can be specified.
prob	A numeric value required when latent_dist = "Mixture". It is a $\pi = \frac{n_1}{N}$ parameter of two-component Gaussian mixture distribution, where n_1 is the estimated number of examinees who belong to the first Gaussian component and N is the total number of examinees (Li, 2021).
d	A numeric value required when latent_dist = "Mixture". It is a $\delta = \frac{\mu_2 - \mu_1}{\bar{\sigma}}$ parameter of two-component Gaussian mixture distribution, where μ_1 is the estimated mean of the first Gaussian component, μ_2 is the estimated mean of the second Gaussian component, and $\bar{\sigma} = 1$ is the standard deviation of the latent distribution (Li, 2021). Without loss of generality, $\mu_2 \geq \mu_1$, thus $\delta \geq 0$, is assumed.
sd_ratio	A numeric value required when latent_dist = "Mixture". It is a $\zeta = \frac{\sigma_2}{\sigma_1}$ parameter of two-component Gaussian mixture distribution, where σ_1 is the estimated standard deviation of the first Gaussian component, σ_2 is the estimated standard deviation of the second Gaussian component (Li, 2021).
m	A numeric value of the overall mean of the latent distribution. The default is 0.
s	A numeric value of the overall standard deviation of the latent distribution. The default is 1.
a_l	A numeric value. The lower bound of item discrimination parameters (*a*).
a_u	A numeric value. The upper bound of item discrimination parameters (*a*).
b_m	A numeric value. The mean of item difficulty parameters (*b*). If unspecified, m is passed on to the value.
b_sd	A numeric value. The standard deviation of item difficulty parameters (*b*). If unspecified, s is passed on to the value.
c_l	A numeric value. The lower bound of item guessing parameters (*c*).
c_u	A numeric value. The lower bound of item guessing parameters (*c*).
categ	A numeric vector of length nitem_P. The *i*th element equals the number of categories of the *i*th polyotomous item.

Value

This function returns a `list` which contains several objects:

`theta`	A vector of ability parameters (θ).
`item_D`	A matrix of dichotomous item parameters.
`initialitem_D`	A matrix that contains initial item parameter values for dichotomous items.
`data_D`	A matrix of dichotomous item responses where rows indicate examinees and columns indicate items.
`item_P`	A matrix of polytomous item parameters.
`initialitem_P`	A matrix that contains initial item parameter values for polytomous items.
`data_P`	A matrix of polytomous item responses where rows indicate examinees and columns indicate items.

Author(s)

Seewoo Li <cu@yonsei.ac.kr>

References

Li, S. (2021). Using a two-component normal mixture distribution as a latent distribution in estimating parameters of item response models. *Journal of Educational Evaluation, 34*(4), 759-789.

Examples

```
# Dichotomous item responses only

Alldata <- DataGeneration(seed = 1,
                          model_D = rep(3, 10),
                          N=500,
                          nitem_D = 10,
                          nitem_P = 0,
                          latent_dist = "2NM",
                          d = 1.664,
                          sd_ratio = 2,
                          prob = 0.3)

data <- Alldata$data_D
item <- Alldata$item_D
initialitem <- Alldata$initialitem_D
theta <- Alldata$theta


# Polytomous item responses only

Alldata <- DataGeneration(seed = 2,
                          N=1000,
                          item_D=NULL,
                          item_P=NULL,
                          theta = NULL,
```

```
                          nitem_D = 0,
                          nitem_P = 10,
                          categ = rep(3:7,each = 2),
                          latent_dist = "2NM",
                          d = 1.664,
                          sd_ratio = 2,
                          prob = 0.3)

data <- Alldata$data_P
item <- Alldata$item_P
initialitem <- Alldata$initialitem_P
theta <- Alldata$theta


# Mixed-format items

Alldata <- DataGeneration(seed = 2,
                         model_D = rep(1:2, each=10),# 1PL model is applied to item #1~10
                                               # and 2PL model is applied to item #11~20.
                          N=1000,
                          nitem_D = 20,
                          nitem_P = 10,
                          categ = rep(3:7,each = 2),# 3 categories for item #21-22,
                                                    # 4 categories for item #23-24,
                                                    # ...,
                                                    # and 7 categories for item #29-30.
                          latent_dist = "2NM",
                          d = 1.664,
                          sd_ratio = 2,
                          prob = 0.3)

DataD <- Alldata$data_D
DataP <- Alldata$data_P
itemD <- Alldata$item_D
itemP <- Alldata$item_P
initialitemD <- Alldata$initialitem_D
initialitemP <- Alldata$initialitem_P
theta <- Alldata$theta
```

dist2 *Re-parameterized two-component normal mixture distribution*

Description

Probability density for the re-parameterized two-component normal mixture distribution.

Usage

```
dist2(x, prob = 0.5, d = 0, sd_ratio = 1, overallmean = 0, overallsd = 1)
```

Arguments

`x`	A numeric vector. The location to evaluate the density function.
`prob`	A numeric value of $\pi = \frac{n_1}{N}$ parameter of two-component Gaussian mixture distribution, where n_1 is the estimated number of examinees who belong to the first Gaussian component and N is the total number of examinees (Li, 2021).
`d`	A numeric value of $\delta = \frac{\mu_2 - \mu_1}{\bar{\sigma}}$ parameter of two-component Gaussian mixture distribution, where μ_1 is the estimated mean of the first Gaussian component, μ_2 is the estimated mean of the second Gaussian component, and $\bar{\sigma}$ is the standard deviation of the latent distribution (Li, 2021). Without loss of generality, $\mu_2 \geq \mu_1$, thus $\delta \geq 0$, is assumed.
`sd_ratio`	A numeric value of $\zeta = \frac{\sigma_2}{\sigma_1}$ parameter of two-component Gaussian mixture distribution, where σ_1 is the estimated standard deviation of the first Gaussian component, σ_2 is the estimated standard deviation of the second Gaussian component (Li, 2021).
`overallmean`	A numeric value of $\bar{\mu}$ that determines the overall mean of two-component Gaussian mixture distribution.
`overallsd`	A numeric value of $\bar{\sigma}$ that determines the overall standard deviation of two-component Gaussian mixture distribution.

Details

The overall mean and overall standard deviation can be expressed with original parameters of two-component Gauss

1) Overall mean ($\bar{\mu}$)

$$\bar{\mu} = \pi\mu_1 + (1 - \pi)\mu_2$$

2) Overall standard deviation ($\bar{\sigma}$)

$$\bar{\sigma} = \sqrt{\pi\sigma_1^2 + (1 - \pi)\sigma_2^2 + \pi(1 - \pi)(\mu_2 - \mu_1)^2}$$

Value

The evaluated probability density value(s).

Author(s)

Seewoo Li `<cu@yonsei.ac.kr>`

References

Li, S. (2021). Using a two-component normal mixture distribution as a latent distribution in estimating parameters of item response models. *Journal of Educational Evaluation, 34*(4), 759-789.

`GHc` *Gauss-Hermite constants*

Description

a `vector` that contains Gauss-Hermite constants

Usage

```
GHc
```

Format

An object of class `numeric` of length 21.

`inform_f_item` *Item information function*

Description

Item information function

Usage

```
inform_f_item(x, test, item, type = NULL)
```

Arguments

`x`	A vector of θ value(s).
`test`	An object returned from an estimation function.
`item`	A numeric value indicating an item. If n is provided, item information is calculated for the nth item.
`type`	A character value for a mixed format test which determines the item type: `"d"` stands for a dichotomous item, and `"p"` stands for a polytomous item.

Value

A vector of item information values of the same length as `x`.

Author(s)

Seewoo Li <cu@yonsei.ac.kr>

`inform_f_test` *Test information function*

Description

Test information function

Usage

```
inform_f_test(x, test)
```

Arguments

`x`	A vector of θ value(s).
`test`	An object returned from an estimation function.

Value

A vector of test information values of the same length as `x`.

Author(s)

Seewoo Li <cu@yonsei.ac.kr>

`IRTest_Dich` *Item and ability parameters estimation for dichotomous items*

Description

This function estimates IRT item and ability parameters when all items are scored dichotomously. Based on Bock & Aitkin's (1981) marginal maximum likelihood and EM algorithm (EM-MML), this function incorporates several latent distribution estimation algorithms which could free the normality assumption on the latent variable. If the normality assumption is violated, application of these latent distribution estimation methods could reflect some features of the unknown true latent distribution, and, thus, could provide more accurate parameter estimates (Li, 2021; Woods & Lin, 2009; Woods & Thissen, 2006).

Usage

```
IRTest_Dich(
  data,
  model = "2PL",
  range = c(-6, 6),
  q = 121,
  initialitem = NULL,
```

```
  ability_method = "EAP",
  latent_dist = "Normal",
  max_iter = 200,
  threshold = 1e-04,
  bandwidth = "SJ-ste",
  h = NULL
)
```

Arguments

`data`	A matrix of item responses where responses are coded as 0 or 1. Rows and columns indicate examinees and items, respectively.
`model`	A vector that represents types of item characteristic functions applied to each item. Insert `1`, `"1PL"`, `"Rasch"`, or `"RASCH"` for one-parameter logistic model, `2`, `"2PL"` for two-parameter logistic model, and `3`, `"3PL"` for three-parameter logistic model. The default is `"2PL"`.
`range`	Range of the latent variable to be considered in the quadrature scheme. The default is from `-6` to `6`: `c(-6, 6)`.
`q`	A numeric value that represents the number of quadrature points. The default value is 121.
`initialitem`	A matrix of initial item parameter values for starting the estimation algorithm
`ability_method`	The ability parameter estimation method. The available options are Expected *a posteriori* (`EAP`) and Maximum Likelihood Estimates (`MLE`). The default is `EAP`.
`latent_dist`	A character string that determines latent distribution estimation method. Insert `"Normal"`, `"normal"`, or `"N"` to assume normal distribution on the latent distribution, `"EHM"` for empirical histogram method (Mislevy, 1984; Mislevy & Bock, 1985), `"Mixture"` or `"2NM"` for the method of two-component Gaussian mixture distribution (Li, 2021; Mislevy, 1984), `"DC"` or `"Davidian"` for Davidian-curve method (Woods & Lin, 2009), `"KDE"` for kernel density estimation method (Li, 2022), and `"LLS"` for log-linear smoothing method (Casabianca, Lewis, 2015). The default value is set to `"Normal"` for the conventional normality assumption on latent distribution.
`max_iter`	A numeric value that determines the maximum number of iterations in the EM-MML. The default value is 200.
`threshold`	A numeric value that determines the threshold of EM-MML convergence. A maximum item parameter change is monitored and compared with the threshold. The default value is 0.0001.
`bandwidth`	A character value is needed when `"KDE"` is used for the latent distribution estimation. This argument determines which bandwidth estimation method is used for `"KDE"`. The default value is `"SJ-ste"`. See `density` for possible options.
`h`	A natural number less than or equal to 10 is needed when `"DC"` is used for the latent distribution estimation. This argument determines the complexity of Davidian curve.

Details

The probabilities for correct response ($u = 1$) in one-, two-, and three-parameter logistic models can be expressed as

1) One-parameter logistic (1PL) model

$$P(u=1|\theta,b) = \frac{\exp(\theta - b)}{1+\exp(\theta - b)}$$

2) Two-parameter logistic (2PL) model

$$P(u=1|\theta,a,b) = \frac{\exp(a(\theta - b))}{1+\exp(a(\theta - b))}$$

3) Three-parameter logistic (3PL) model

$$P(u=1|\theta,a,b,c) = c + (1-c)\frac{\exp(a(\theta - b))}{1+\exp(a(\theta - b))}$$

The estimated latent distribution for each of the latent distribution estimation method can be expressed as follows;

1) Empirical histogram method

$$P(\theta = X_k) = A(X_k)$$

where $k = 1, 2, ..., q$, X_k is the location of the kth quadrature point, and $A(X_k)$ is a value of probability mass function evaluated at X_k. Empirical histogram method thus has $q-1$ parameters.

2) Two-component Gaussian mixture distribution

$$P(\theta = X) = \pi\phi(X;\mu_1,\sigma_1) + (1-\pi)\phi(X;\mu_2,\sigma_2)$$

where $\phi(X;\mu,\sigma)$ is the value of a Gaussian component with mean μ and standard deviation σ evaluated at X.

3) Davidian curve method

$$P(\theta = X) = \left\{\sum_{\lambda=0}^{h} m_\lambda X^\lambda\right\}^2 \phi(X;0,1)$$

where h corresponds to the argument `h` and determines the degree of the polynomial.

4) Kernel density estimation method

$$P(\theta = X) = \frac{1}{Nh}\sum_{j=1}^{N} K\left(\frac{X-\theta_j}{h}\right)$$

where N is the number of examinees, θ_j is jth examinee's ability parameter, h is the bandwidth which corresponds to the argument `bandwidth`, and $K(\cdot)$ is a kernel function. The Gaussian kernel is used in this function.

5) Log-linear smoothing method

$$P(\theta = X_q) = \exp\left(\beta_0 + \sum_{m=1}^{h} \beta_m X_q^m\right)$$

where h is the hyper parameter which determines the smoothness of the density, and θ can take total Q finite values $(X_1, \ldots, X_q, \ldots, X_Q)$.

Value

This function returns a list which contains several objects:

- par_est: The item parameter estimates.
- se: The asymptotic standard errors for item parameter estimates.
- fk: The estimated frequencies of examinees at each quadrature points.
- iter: The number of EM-MML iterations required for the convergence.
- quad: The location of quadrature points.
- diff: The final value of the monitored maximum item parameter change.
- Ak: The estimated discrete latent distribution. It is discrete (i.e., probability mass function) since quadrature scheme of EM-MML is used.
- Pk: The posterior probabilities for each examinees at each quadrature points.
- theta: The estimated ability parameter values. If ability_method = "MLE", and if an examinee answers all or none of the items correctly, the function returns $\pm$Inf.
- theta_se: The asymptotic standard errors of ability parameter estimates. Available only when ability_method = "MLE". If an examinee answers all or none of the items correctly, the function returns NA.
- logL: The deviance (i.e., -2*log*L).
- density_par: The estimated density parameters. If latent_dist = "2NM", prob is the estimated $\pi = \frac{n_1}{N}$ parameter of two-component Gaussian mixture distribution, where n_1 is the estimated number of examinees who belong to the first Gaussian component and N is the total number of examinees; d is the estimated $\delta = \frac{\mu_2 - \mu_1}{\bar{\sigma}}$ parameter of two-component Gaussian mixture distribution, where μ_1 is the estimated mean of the first Gaussian component, μ_2 is the estimated mean of the second Gaussian component, and $\bar{\sigma} = 1$ is the standard deviation of the latent distribution; and sd_ratio is the estimated $\zeta = \frac{\sigma_2}{\sigma_1}$ parameter of two-component Gaussian mixture distribution, where σ_1 is the estimated standard deviation of the first Gaussian component, σ_2 is the estimated standard deviation of the second Gaussian component (Li, 2021). Without loss of generality, $\mu_2 \geq \mu_1$, thus $\delta \geq 0$, is assumed.
- Options: A replication of input arguments and other information.

Author(s)

Seewoo Li <cu@yonsei.ac.kr>

References

Bock, R. D., & Aitkin, M. (1981). Marginal maximum likelihood estimation of item parameters: Application of an EM algorithm. *Psychometrika, 46*(4), 443-459.

Casabianca, J. M., & Lewis, C. (2015). IRT item parameter recovery with marginal maximum likelihood estimation using loglinear smoothing models. *Journal of Educational and Behavioral Statistics, 40*(6), 547-578.

Li, S. (2021). Using a two-component normal mixture distribution as a latent distribution in estimating parameters of item response models. *Journal of Educational Evaluation, 34*(4), 759-789.

Li, S. (2022). *The effect of estimating latent distribution using kernel density estimation method on the accuracy and efficiency of parameter estimation of item response models* [Master's thesis, Yonsei University, Seoul]. Yonsei University Library.

Mislevy, R. J. (1984). Estimating latent distributions. *Psychometrika, 49*(3), 359-381.

Mislevy, R. J., & Bock, R. D. (1985). Implementation of the EM algorithm in the estimation of item parameters: The BILOG computer program. In D. J. Weiss (Ed.). *Proceedings of the 1982 item response theory and computerized adaptive testing conference* (pp. 189-202). University of Minnesota, Department of Psychology, Computerized Adaptive Testing Conference.

Woods, C. M., & Lin, N. (2009). Item response theory with estimation of the latent density using Davidian curves. *Applied Psychological Measurement, 33*(2), 102-117.

Woods, C. M., & Thissen, D. (2006). Item response theory with estimation of the latent population distribution using spline-based densities. *Psychometrika, 71*(2), 281-301.

Examples

```
## Not run:
# A preparation of dichotomous item response data

data <- DataGeneration(seed = 1,
                       model_D = rep(1, 10),
                       N=500,
                       nitem_D = 10,
                       nitem_P = 0,
                       latent_dist = "2NM",
                       d = 1.664,
                       sd_ratio = 2,
                       prob = 0.3)$data_D


# Analysis

M1 <- IRTest_Dich(data)

## End(Not run)
```

IRTest_Mix — *Item and ability parameters estimation for a mixed-format item response data*

Description

This function estimates IRT item and ability parameters when a test consists of mixed-format items (i.e., a combination of dichotomous and polytomous items). In educational context, the combination of these two item formats takes an advantage; Dichotomous item format expedites scoring and is conducive to cover broad domain, while Polytomous item format (e.g., free response item) encourages students to exert complex cognitive skills (Lee et al., 2020). Based on Bock & Aitkin's (1981) marginal maximum likelihood and EM algorithm (EM-MML), this function incorporates several latent distribution estimation algorithms which could free the normality assumption on the latent variable. If the normality assumption is violated, application of these latent distribution estimation methods could reflect some features of the unknown true latent distribution, and, thus, could provide more accurate parameter estimates (Li, 2021; Woods & Lin, 2009; Woods & Thissen, 2006).

Usage

```
IRTest_Mix(
  data_D,
  data_P,
  model_D = "2PL",
  model_P = "GPCM",
  range = c(-6, 6),
  q = 121,
  initialitem_D = NULL,
  initialitem_P = NULL,
  ability_method = "EAP",
  latent_dist = "Normal",
  max_iter = 200,
  threshold = 1e-04,
  bandwidth = "SJ-ste",
  h = NULL
)
```

Arguments

data_D	A matrix of item responses where responses are coded as 0 or 1. Rows and columns indicate examinees and items, respectively.
data_P	A matrix of polytomous item responses where responses are coded as 0, 1, ..., or m for an m+1 category item. Rows and columns indicate examinees and items, respectively.
model_D	A vector dichotomous item response models. Insert 1, "1PL", "Rasch", or "RASCH" for one-parameter logistic model, 2, "2PL" for two-parameter logistic model, and 3, "3PL" for three-parameter logistic model. The default is "2PL".
model_P	A character value of an item response model. Currently, PCM, GPCM, and GRM are available. The default is "GPCM".
range	Range of the latent variable to be considered in the quadrature scheme. The default is from -6 to 6: c(-6, 6).
q	A numeric value that represents the number of quadrature points. The default value is 121.
initialitem_D	A matrix of initial dichotomous item parameter values for starting the estimation algorithm.
initialitem_P	A matrix of initial polytomous item parameter values for starting the estimation algorithm.
ability_method	The ability parameter estimation method. The available options are Expected *a posteriori* (EAP) and Maximum Likelihood Estimates (MLE). The default is EAP.

`latent_dist` A character string that determines latent distribution estimation method. Insert `"Normal"`, `"normal"`, or `"N"` to assume normal distribution on the latent distribution, `"EHM"` for empirical histogram method (Mislevy, 1984; Mislevy & Bock, 1985), `"Mixture"` or `"2NM"` for the method of two-component Gaussian mixture distribution (Li, 2021; Mislevy, 1984), `"DC"` or `"Davidian"` for Davidian-curve method (Woods & Lin, 2009), `"KDE"` for kernel density estimation method (Li, 2022), and `"LLS"` for log-linear smoothing method (Casabianca, Lewis, 2015). The default value is set to `"Normal"` for the conventional normality assumption on latent distribution.

`max_iter` A numeric value that determines the maximum number of iterations in the EM-MML. The default value is 200.

`threshold` A numeric value that determines the threshold of EM-MML convergence. A maximum item parameter change is monitored and compared with the threshold. The default value is 0.0001.

`bandwidth` A character value is needed when `"KDE"` is used for the latent distribution estimation. This argument determines which bandwidth estimation method is used for `"KDE"`. The default value is `"SJ-ste"`. See `density` for possible options.

`h` A natural number less than or equal to 10 is needed when `"DC"` is used for the latent distribution estimation. This argument determines the complexity of Davidian curve.

Details

Dichotomous: The probabilities for correct response $(u = 1)$ **in one-, two-, and three-parameter logistic models can b**

1) One-parameter logistic (1PL) model

$$P(u = 1|\theta, b) = \frac{\exp(\theta - b)}{1 + \exp(\theta - b)}$$

2) Two-parameter logistic (2PL) model

$$P(u = 1|\theta, a, b) = \frac{\exp(a(\theta - b))}{1 + \exp(a(\theta - b))}$$

3) Three-parameter logistic (3PL) model

$$P(u = 1|\theta, a, b, c) = c + (1 - c)\frac{\exp(a(\theta - b))}{1 + \exp(a(\theta - b))}$$

Polytomous: The probability for scoring k **(i.e.,** $u = k; k = 0, 1, ..., m; m \geq 2$**)can be expressed as follows;**

1) partial credit model (PCM)

$$P(u = 0|\theta, b_1, ..., b_m) = \frac{1}{1 + \sum_{c=1}^{m} \exp\left[\sum_{v=1}^{c} a(\theta - b_v)\right]}$$

$$P(u = 1|\theta, b_1, ..., b_m) = \frac{\exp(\theta - b_1)}{1 + \sum_{c=1}^{m} \exp\left[\sum_{v=1}^{c} \theta - b_v\right]}$$

$$\vdots$$

$$P(u=m|\theta,b_1,...,b_m)=\frac{\exp\left[\sum_{v=1}^{m}\theta-b_v\right]}{1+\sum_{c=1}^{m}\exp\left[\sum_{v=1}^{c}\theta-b_v\right]}$$

2) generalized partial credit model (GPCM)

$$P(u=0|\theta,a,b_1,...,b_m)=\frac{1}{1+\sum_{c=1}^{m}\exp\left[\sum_{v=1}^{c}a(\theta-b_v)\right]}$$

$$P(u=1|\theta,a,b_1,...,b_m)=\frac{\exp\left(a(\theta-b_1)\right)}{1+\sum_{c=1}^{m}\exp\left[\sum_{v=1}^{c}a(\theta-b_v)\right]}$$

$$\vdots$$

$$P(u=m|\theta,a,b_1,...,b_m)=\frac{\exp\left[\sum_{v=1}^{m}a(\theta-b_v)\right]}{1+\sum_{c=1}^{m}\exp\left[\sum_{v=1}^{c}a(\theta-b_v)\right]}$$

The estimated latent distribution for each of the latent distribution estimation method can be expressed as follows;

1) Empirical histogram method

$$P(\theta=X_k)=A(X_k)$$

where $k=1,2,...,q$, X_k is the location of the kth quadrature point, and $A(X_k)$ is a value of probability mass function evaluated at X_k. Empirical histogram method thus has $q-1$ parameters.

2) Two-component Gaussian mixture distribution

$$P(\theta=X)=\pi\phi(X;\mu_1,\sigma_1)+(1-\pi)\phi(X;\mu_2,\sigma_2)$$

where $\phi(X;\mu,\sigma)$ is the value of a Gaussian component with mean μ and standard deviation σ evaluated at X.

3) Davidian curve method

$$P(\theta=X)=\left\{\sum_{\lambda=0}^{h}m_\lambda X^\lambda\right\}^2\phi(X;0,1)$$

where h corresponds to the argument `h` and determines the degree of the polynomial.

4) Kernel density estimation method

$$P(\theta=X)=\frac{1}{Nh}\sum_{j=1}^{N}K\left(\frac{X-\theta_j}{h}\right)$$

where N is the number of examinees, θ_j is jth examinee's ability parameter, h is the bandwidth which corresponds to the argument `bw`, and $K(\bullet)$ is a kernel function. The Gaussian kernel is used in this function.

5) Log-linear smoothing method

$$P(\theta=X_q)=\exp\left(\beta_0+\sum_{m=1}^{h}\beta_m X_q^m\right)$$

where h is the hyper parameter which determines the smoothness of the density, and θ can take total Q finite values ($X_1,\ldots,X_q,\ldots,X_Q$).

Value

This function returns a `list` which contains several objects:

`par_est`	The list item parameter estimates. The first object of `par_est` is the matrix of item parameter estimates for dichotomous items, and The second object is the matrix of item parameter estimates for polytomous items.
`se`	The standard errors for item parameter estimates. The first object of `se` is the matrix of standard errors for dichotomous items, and The second object is the matrix of standard errors for polytomous items.
`fk`	The estimated frequencies of examinees at each quadrature points.
`iter`	The number of EM-MML iterations required for the convergence.
`quad`	The location of quadrature points.
`diff`	The final value of the monitored maximum item parameter change.
`Ak`	The estimated discrete latent distribution. It is discrete (i.e., probability mass function) since quadrature scheme of EM-MML is used.
`Pk`	The posterior probabilities for each examinees at each quadrature points.
`theta`	The estimated ability parameter values.
`theta_se`	The asymptotic standard errors of ability parameter estimates. Available only when `ability_method = "MLE"`. If an examinee answers all or none of the items correctly, the function returns `NA`.
`logL`	The deviance (i.e., -2*log*L).
`density_par`	The estimated density parameters. If `latent_dist = "2NM"`, `prob` is the estimated $\pi = \frac{n_1}{N}$ parameter of two-component Gaussian mixture distribution, where n_1 is the estimated number of examinees who belong to the first Gaussian component and N is the total number of examinees; `d` is the estimated $\delta = \frac{\mu_2 - \mu_1}{\bar{\sigma}}$ parameter of two-component Gaussian mixture distribution, where μ_1 is the estimated mean of the first Gaussian component, μ_2 is the estimated mean of the second Gaussian component, and $\bar{\sigma} = 1$ is the standard deviation of the latent distribution; and `sd_ratio` is the estimated $\zeta = \frac{\sigma_2}{\sigma_1}$ parameter of two-component Gaussian mixture distribution, where σ_1 is the estimated standard deviation of the first Gaussian component, σ_2 is the estimated standard deviation of the second Gaussian component (Li, 2021). Without loss of generality, $\mu_2 \geq \mu_1$, thus $\delta \geq 0$, is assumed.
`Options`	A replication of input arguments and other information.

Author(s)

Seewoo Li `<cu@yonsei.ac.kr>`

References

Bock, R. D., & Aitkin, M. (1981). Marginal maximum likelihood estimation of item parameters: Application of an EM algorithm. *Psychometrika, 46*(4), 443-459.

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Mislevy, R. J., & Bock, R. D. (1985). Implementation of the EM algorithm in the estimation of item parameters: The BILOG computer program. In D. J. Weiss (Ed.). *Proceedings of the 1982 item response theory and computerized adaptive testing conference* (pp. 189-202). University of Minnesota, Department of Psychology, Computerized Adaptive Testing Conference.

Woods, C. M., & Lin, N. (2009). Item response theory with estimation of the latent density using Davidian curves. *Applied Psychological Measurement, 33*(2), 102-117.

Woods, C. M., & Thissen, D. (2006). Item response theory with estimation of the latent population distribution using spline-based densities. *Psychometrika, 71*(2), 281-301.

Examples

```
## Not run:
# A preparation of mixed-format item response data

Alldata <- DataGeneration(seed = 2,
                          model_D = rep(1:2, each=3),# 1PL model is applied to item #1~10
                                                      # and 2PL model is applied to item #11~20.
                           N=1000,
                           nitem_D = 6,
                           nitem_P = 5,
                           categ = rep(3:7,each = 1),# 3 categories for item #21-22,
                                                     # 4 categories for item #23-24,
                                                     # ...,
                                                     # and 7 categories for item #29-30.
                           latent_dist = "2NM",
                           d = 1.664,
                           sd_ratio = 2,
                           prob = 0.3)

DataD <- Alldata$data_D   # item response data for the dichotomous items
DataP <- Alldata$data_P   # item response data for the polytomous items

# Analysis

M1 <- IRTest_Mix(DataD, DataP)

## End(Not run)
```

IRTest_Poly *Item and ability parameters estimation for polytomous items*

Description

This function estimates IRT item and ability parameters when all items are scored polytomously. Based on Bock & Aitkin's (1981) marginal maximum likelihood and EM algorithm (EM-MML), this function incorporates several latent distribution estimation algorithms which could free the normality assumption on the latent variable. If the normality assumption is violated, application of these latent distribution estimation methods could reflect some features of the unknown true latent distribution, and, thus, could provide more accurate parameter estimates (Li, 2021; Woods & Lin, 2009; Woods & Thissen, 2006). Only generalized partial credit model (GPCM) is currently available.

Usage

```
IRTest_Poly(
  data,
  model = "GPCM",
  range = c(-6, 6),
  q = 121,
  initialitem = NULL,
  ability_method = "EAP",
  latent_dist = "Normal",
  max_iter = 200,
  threshold = 1e-04,
  bandwidth = "SJ-ste",
  h = NULL
)
```

Arguments

`data`	A matrix of item responses where responses are coded as `0, 1, ..., m` for an `m+1` category item. Rows and columns indicate examinees and items, respectively.
`model`	A character value that represents the type of a item characteristic function applied to the items. Currently, `PCM`, `GPCM`, and `GRM` are available. The default is `"GPCM"`.
`range`	Range of the latent variable to be considered in the quadrature scheme. The default is from `-6` to `6`: `c(-6, 6)`.
`q`	A numeric value that represents the number of quadrature points. The default value is 121.
`initialitem`	A matrix of initial item parameter values for starting the estimation algorithm. This matrix determines the number of categories for each item.
`ability_method`	The ability parameter estimation method. The available options are Expected *a posteriori* (`EAP`) and Maximum Likelihood Estimates (`MLE`). The default is `EAP`.

latent_dist
: A character string that determines latent distribution estimation method. Insert "Normal", "normal", or "N" to assume normal distribution on the latent distribution, "EHM" for empirical histogram method (Mislevy, 1984; Mislevy & Bock, 1985), "Mixture" or "2NM" for the method of two-component Gaussian mixture distribution (Li, 2021; Mislevy, 1984), "DC" or "Davidian" for Davidian-curve method (Woods & Lin, 2009), "KDE" for kernel density estimation method (Li, 2022), and "LLS" for log-linear smoothing method (Casabianca, Lewis, 2015). The default value is set to "Normal" for the conventional normality assumption on latent distribution.

max_iter
: A numeric value that determines the maximum number of iterations in the EM-MML. The default value is 200.

threshold
: A numeric value that determines the threshold of EM-MML convergence. A maximum item parameter change is monitored and compared with the threshold. The default value is 0.0001.

bandwidth
: A character value is needed when "KDE" is used for the latent distribution estimation. This argument determines which bandwidth estimation method is used for "KDE". The default value is "SJ-ste". See density for possible options.

h
: A natural number less than or equal to 10 is needed when "DC" is used for the latent distribution estimation. This argument determines the complexity of Davidian curve.

Details

The probability for scoring k (i.e., $u=k; k=0,1,...,m; m \geq 2$)can be expressed as follows;

1) partial credit model (PCM)

$$P(u=0|\theta, b_1, ..., b_m) = \frac{1}{1+\sum_{c=1}^{m}\exp{[\sum_{v=1}^{c}a(\theta-b_v)]}}$$

$$P(u=1|\theta, b_1, ..., b_m) = \frac{\exp{(\theta-b_1)}}{1+\sum_{c=1}^{m}\exp{[\sum_{v=1}^{c}\theta-b_v]}}$$

$$\vdots$$

$$P(u=m|\theta, b_1, ..., b_m) = \frac{\exp{[\sum_{v=1}^{m}\theta-b_v]}}{1+\sum_{c=1}^{m}\exp{[\sum_{v=1}^{c}\theta-b_v]}}$$

2) generalized partial credit model (GPCM)

$$P(u=0|\theta, a, b_1, ..., b_m) = \frac{1}{1+\sum_{c=1}^{m}\exp{[\sum_{v=1}^{c}a(\theta-b_v)]}}$$

$$P(u=1|\theta, a, b_1, ..., b_m) = \frac{\exp{(a(\theta-b_1))}}{1+\sum_{c=1}^{m}\exp{[\sum_{v=1}^{c}a(\theta-b_v)]}}$$

$$\vdots$$

$$P(u=m|\theta, a, b_1, ..., b_m) = \frac{\exp{[\sum_{v=1}^{m}a(\theta-b_v)]}}{1+\sum_{c=1}^{m}\exp{[\sum_{v=1}^{c}a(\theta-b_v)]}}$$

The estimated latent distribution for each of the latent distribution estimation method can be expressed as follows;

1) Empirical histogram method

$$P(\theta = X_k) = A(X_k)$$

where $k = 1, 2, ..., q$, X_k is the location of the kth quadrature point, and $A(X_k)$ is a value of probability mass function evaluated at X_k. Empirical histogram method thus has $q - 1$ parameters.

2) Two-component Gaussian mixture distribution

$$P(\theta = X) = \pi\phi(X; \mu_1, \sigma_1) + (1 - \pi)\phi(X; \mu_2, \sigma_2)$$

where $\phi(X; \mu, \sigma)$ is the value of a Gaussian component with mean μ and standard deviation σ evaluated at X.

3) Davidian curve method

$$P(\theta = X) = \left\{ \sum_{\lambda=0}^{h} m_\lambda X^\lambda \right\}^2 \phi(X; 0, 1)$$

where h corresponds to the argument `h` and determines the degree of the polynomial.

4) Kernel density estimation method

$$P(\theta = X) = \frac{1}{Nh} \sum_{j=1}^{N} K\left(\frac{X - \theta_j}{h} \right)$$

where N is the number of examinees, θ_j is jth examinee's ability parameter, h is the bandwidth which corresponds to the argument `bw`, and $K(\bullet)$ is a kernel function. The Gaussian kernel is used in this function.

5) Log-linear smoothing method

$$P(\theta = X_q) = \exp\left(\beta_0 + \sum_{m=1}^{h} \beta_m X_q^m \right)$$

where h is the hyper parameter which determines the smoothness of the density, and θ can take total Q finite values ($X_1, \ldots, X_q, \ldots, X_Q$).

Value

This function returns a `list` which contains several objects:

`par_est`	The item parameter estimates.
`se`	The standard errors for item parameter estimates.
`fk`	The estimated frequencies of examinees at each quadrature points.
`iter`	The number of EM-MML iterations required for the convergence.
`quad`	The location of quadrature points.
`diff`	The final value of the monitored maximum item parameter change.
`Ak`	The estimated discrete latent distribution. It is discrete (i.e., probability mass function) since quadrature scheme of EM-MML is used.

Pk	The posterior probabilities for each examinees at each quadrature points.
theta	The estimated ability parameter values.
theta_se	The asymptotic standard errors of ability parameter estimates. Available only when ability_method = "MLE". If an examinee answers all or none of the items correctly, the function returns NA.
logL	The deviance (i.e., -2logL).
density_par	The estimated density parameters. If latent_dist = "2NM", prob is the estimated $\pi = \frac{n_1}{N}$ parameter of two-component Gaussian mixture distribution, where n_1 is the estimated number of examinees who belong to the first Gaussian component and N is the total number of examinees; d is the estimated $\delta = \frac{\mu_2 - \mu_1}{\bar{\sigma}}$ parameter of two-component Gaussian mixture distribution, where μ_1 is the estimated mean of the first Gaussian component, μ_2 is the estimated mean of the second Gaussian component, and $\bar{\sigma} = 1$ is the standard deviation of the latent distribution; and sd_ratio is the estimated $\zeta = \frac{\sigma_2}{\sigma_1}$ parameter of two-component Gaussian mixture distribution, where σ_1 is the estimated standard deviation of the first Gaussian component, σ_2 is the estimated standard deviation of the second Gaussian component (Li, 2021). Without loss of generality, $\mu_2 \geq \mu_1$, thus $\delta \geq 0$, is assumed.
Options	A replication of input arguments and other information.

Author(s)

Seewoo Li <cu@yonsei.ac.kr>

References

Bock, R. D., & Aitkin, M. (1981). Marginal maximum likelihood estimation of item parameters: Application of an EM algorithm. *Psychometrika, 46*(4), 443-459.

Casabianca, J. M., & Lewis, C. (2015). IRT item parameter recovery with marginal maximum likelihood estimation using loglinear smoothing models. *Journal of Educational and Behavioral Statistics, 40*(6), 547-578.

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Li, S. (2022). *The effect of estimating latent distribution using kernel density estimation method on the accuracy and efficiency of parameter estimation of item response models* [Master's thesis, Yonsei University, Seoul]. Yonsei University Library.

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Woods, C. M., & Lin, N. (2009). Item response theory with estimation of the latent density using Davidian curves. *Applied Psychological Measurement, 33*(2), 102-117.

Woods, C. M., & Thissen, D. (2006). Item response theory with estimation of the latent population distribution using spline-based densities. *Psychometrika, 71*(2), 281-301.

Examples

```
## Not run:
# A preparation of dichotomous item response data

data <- DataGeneration(seed = 1,
                       model_P = "GPCM",
                       categ = rep(3:4, each = 4),
                       N=1000,
                       nitem_D = 0,
                       nitem_P = 8,
                       latent_dist = "2NM",
                       d = 1.414,
                       sd_ratio = 2,
                       prob = 0.5)$data_P

# Analysis

M1 <- IRTest_Poly(data)

## End(Not run)
```

`item_fit` *Item fit diagnostics*

Description

This function analyses and reports item-fit test results.

Usage

```
item_fit(x, bins = 10, bin.center = "mean")
```

Arguments

x	A model fit object from either `IRTest_Dich`, `IRTest_Poly`, or `IRTest_Mix`.
bins	The number of bins to be used for calculating the statistics. Following Yen's Q_1 (1981) , the default is 10.
bin.center	A method for calculating the center of each bin. Following Yen's Q_1 (1981) , the default is `"mean"`. Use `"median"` for Bock's χ^2 (1960).

Details

Bock's χ^2 (1960) or Yen's Q_1 (1981) is currently available.

Value

This function returns a `matrix` of item-fit test results.

Author(s)

Seewoo Li <cu@yonsei.ac.kr>

References

Bock, R.D. (1960), Methods and applications of optimal scaling. Chapel Hill, NC: L.L. Thurstone Psychometric Laboratory.

Yen, W. M. (1981). Using simulation results to choose a latent trait model. *Applied Psychological Measurement, 5*(2), 245–262.

---

latent_distribution *Latent density function*

Description

Density function of the estimated latent distribution with mean and standard deviation equal to 0 and 1, respectively.

Usage

```
latent_distribution(x, model.fit)
```

Arguments

x	A numeric vector. Value(s) in the $theta$ scale to evaluate the PDF.
model.fit	An object returned from an estimation function.

Value

The evaluated values of PDF, a length of which equals to that of x.

Examples

```
## Not run:
# Data generation and model fitting

data <- DataGeneration(seed = 1,
                       model_P = "GPCM",
                       N=1000,
                       nitem_D = 0,
                       nitem_P = 10,
                       categ = rep(5,10),
                       latent_dist = "2NM",
                       d = 1.664,
                       sd_ratio = 2,
                       prob = 0.3)$data_P

M1 <- IRTest_Poly(data = data, latent_dist = "KDE")
```

```
# Plotting the latent distribution
ggplot()+
  stat_function(fun=latent_distribution, args=list(M1))+
  lims(x=c(-6,6))

## End(Not run)
```

`original_par_2GM` *Recovering original parameters of two-component Gaussian mixture distribution from re-parameterized parameters*

Description

Recovering original parameters of two-component Gaussian mixture distribution from re-parameterized parameters

Usage

```
original_par_2GM(
  prob = 0.5,
  d = 0,
  sd_ratio = 1,
  overallmean = 0,
  overallsd = 1
)
```

Arguments

`prob`	A numeric value of $\pi = \frac{n_1}{N}$ parameter of two-component Gaussian mixture distribution, where n_1 is the estimated number of examinees who belong to the first Gaussian component and N is the total number of examinees (Li, 2021).
`d`	A numeric value of $\delta = \frac{\mu_2 - \mu_1}{\bar{\sigma}}$ parameter of two-component Gaussian mixture distribution, where μ_1 is the estimated mean of the first Gaussian component, μ_2 is the estimated mean of the second Gaussian component, and $\bar{\sigma}$ is the standard deviation of the latent distribution (Li, 2021). Without loss of generality, $\mu_2 \geq \mu_1$, thus $\delta \geq 0$, is assumed.
`sd_ratio`	A numeric value of $\zeta = \frac{\sigma_2}{\sigma_1}$ parameter of two-component Gaussian mixture distribution, where σ_1 is the estimated standard deviation of the first Gaussian component, σ_2 is the estimated standard deviation of the second Gaussian component (Li, 2021).
`overallmean`	A numeric value of $\bar{\mu}$ that determines the overall mean of two-component Gaussian mixture distribution.
`overallsd`	A numeric value of $\bar{\sigma}$ that determines the overall standard deviation of two-component Gaussian mixture distribution.

Details

The original two-component Gaussian mixture distribution

$$f(x) = \pi \times \phi(x|\mu_1, \sigma_1) + (1 - \pi) \times \phi(x|\mu_2, \sigma_2)$$

, where ϕ is a Gaussian component.

The re-parameterized two-component Gaussian mixture distribution

$$f(x) = 2GM(x|\pi, \delta, \zeta, \bar{\mu}, \bar{\sigma})$$

, where $\bar{\mu}$ is overall mean and $\bar{\sigma}$ is overall standard deviation of the distribution.

The original parameters of two-component Gaussian mixture distribution can be retrieved as follows;

1) Mean of the first Gaussian component (`m1`).

$$\mu_1 = -(1 - \pi)\delta\bar{\sigma} + \bar{\mu}$$

2) Mean of the second Gaussian component (`m2`).

$$\mu_2 = \pi\delta\bar{\sigma} + \bar{\mu}$$

3) Standard deviation of the first Gaussian component (`s1`).

$$\sigma_1^2 = \bar{\sigma}^2 \left(\frac{1 - \pi(1 - \pi)\delta^2}{\pi + (1 - \pi)\zeta^2} \right)$$

4) Standard deviation of the second Gaussian component (`s2`).

$$\sigma_2^2 = \bar{\sigma}^2 \left(\frac{1 - \pi(1 - \pi)\delta^2}{\frac{1}{\zeta^2}\pi + (1 - \pi)} \right) = \zeta^2\sigma_1^2$$

Value

This function returns a vector of length 4: `c(m1,m2,s1,s2)`.

`m1`	The location parameter (mean) of the first Gaussian component.
`m2`	The location parameter (mean) of the second Gaussian component.
`s1`	The scale parameter (standard deviation) of the first Gaussian component.
`s2`	The scale parameter (standard deviation) of the second Gaussian component.

Author(s)

Seewoo Li `<cu@yonsei.ac.kr>`

References

Li, S. (2021). Using a two-component normal mixture distribution as a latent distribution in estimating parameters of item response models. *Journal of Educational Evaluation, 34*(4), 759-789.

plot.irtest — *Plotting the estimated latent distribution*

Description

This function draws a plot of the estimated latent distribution (the population distribution of the latent variable).

Usage

```
## S3 method for class 'irtest'
plot(x, ...)
```

Arguments

x	A class == "irtest" object obtained from either IRTest_Dich, IRTest_Poly, or IRTest_Mix.
...	Other argument(s) passed on to draw a plot of an estimated latent distribution. Arguments are passed on to stat_function, if the distribution estimation method is the one using two-component normal mixture distribution (i.e., latent_dist == "Mixture" or "2NM") or the normal distribution (i.e., latent_dist == "N", "normal", or "Normal"). Otherwise, they are passed on to geom_line.

Value

A plot of estimated latent distribution.

Author(s)

Seewoo Li <cu@yonsei.ac.kr>

Examples

```
## Not run:
# Data generation and model fitting

data <- DataGeneration(seed = 1,
                       #model_D = rep(1, 10),
                       N=1000,
                       nitem_D = 0,
                       nitem_P = 8,
                       categ = rep(3:4,each = 4),
                       latent_dist = "2NM",
                       d = 1.664,
                       sd_ratio = 2,
                       prob = 0.3)$data_P

M1 <- IRTest_Poly(data = data, latent_dist = "KDE")
```

```
# Plotting the latent distribution

plot(x=M1, linewidth = 1, color = 'red') +
  ggplot2::lims(x = c(-6, 6), y = c(0, .5))

## End(Not run)
```

---

plot_item *Plot of item response function*

Description

This function draws a item response function of an item of the fitted model.

Usage

```
plot_item(x, item.number, type = NULL)
```

Arguments

x	A model fit object from either IRTest_Dich, IRTest_Poly, or IRTest_Mix.
item.number	A numeric value indicating the item number.
type	Type of an item being either "d" (dichotomous item) or "p" (polytomous item); item.number=1, type="d" indicates the first dichotomous item. This value will be used only when a model fit of mixed-format data is passed onto the function.

Value

This function returns a plot of item response function.

Author(s)

Seewoo Li <cu@yonsei.ac.kr>

print.irtest *Print of the result*

Description

This function prints the summarized information.

Usage

```
## S3 method for class 'irtest'
print(x, ...)
```

Arguments

x	An object returned from summary.irtest.
...	Additional arguments (currently nonfunctioning).

Value

Printed texts on the console recommending the usage of summary function and the direct access to the details using "$" sign.

Author(s)

Seewoo Li <cu@yonsei.ac.kr>

Examples

```
## Not run:
data <- DataGeneration(seed = 1,
                       #model_D = rep(1, 10),
                       N=1000,
                       nitem_D = 0,
                       nitem_P = 8,
                       categ = rep(3:4,each = 4),
                       latent_dist = "2NM",
                       d = 1.664,
                       sd_ratio = 2,
                       prob = 0.3)$data_P

M1 <- IRTest_Poly(data = data, latent_dist = "KDE")

M1

## End(Not run)
```

print.irtest_summary *Print of the summary*

Description

This function prints the summarized information.

Usage

```
## S3 method for class 'irtest_summary'
print(x, ...)
```

Arguments

x	An object returned from summary.irtest.
...	Additional arguments (currently nonfunctioning).

Value

Printed summarized texts on the console.

Author(s)

Seewoo Li <cu@yonsei.ac.kr>

Examples

```
## Not run:
Alldata <- DataGeneration(seed = 1,
                          #model_D = rep(1, 10),
                          N=1000,
                          nitem_D = 0,
                          nitem_P = 8,
                          categ = rep(3:4,each = 4),
                          latent_dist = "2NM",
                          d = 1.664,
                          sd_ratio = 2,
                          prob = 0.3)
data <- Alldata$data_P
item <- Alldata$item_P
initialitem <- Alldata$initialitem_P
theta <- Alldata$theta
M1 <- IRTest_Poly(initialitem = initialitem,
                  data = data,
                  model = "GPCM",
                  latent_dist = "Mixture",
                  max_iter = 200,
                  threshold = .001,
                  )
```

```
summary(M1)

## End(Not run)
```

`recategorize` *Recategorization of data using a new categorization scheme*

Description

With a recategorization scheme as an input, this function implements recategorization for the input data.

Usage

```
recategorize(data, new_cat)
```

Arguments

`data`	An item response matrix
`new_cat`	A list of a new categorization scheme

Value

Recategorized data

`reliability` *Marginal reliability coefficient of IRT*

Description

Marginal reliability coefficient of IRT

Usage

```
reliability(x)
```

Arguments

`x`	A model fit object from either `IRTest_Dich`, `IRTest_Poly`, or `IRTest_Mix`.

Details

Reliability coefficient on summed-score scale In accordance with the concept of *reliability* in classical test theory (CTT), this function calculates the IRT reliability coefficient.

The basic concept and formula of the reliability coefficient can be expressed as follows (Kim, Feldt, 2010):

An observed score of Item i, X_i, is decomposed as the sum of a true score T_i and an error e_i. Then, with the assumption of $\sigma_{T_i e_j} = \sigma_{e_i e_j} = 0$, the reliability coefficient of a test is defined as;

$$\rho_{TX} = \rho_{XX'} = \frac{\sigma_T^2}{\sigma_X^2} = \frac{\sigma_T^2}{\sigma_T^2 + \sigma_e^2} = 1 - \frac{\sigma_e^2}{\sigma_X^2}$$

Reliability coefficient on θ scale For the coefficient on the θ scale, this function calculates the parallel-forms reliability (Green et al., 1984; Kim, 2012):

$$\rho_{\hat{\theta}\hat{\theta}'} = \frac{\sigma^2_{E\left(\hat{\theta}|\theta\right)}}{\sigma^2_{E\left(\hat{\theta}|\theta\right)} + E\left(\sigma^2_{\hat{\theta}|\theta}\right)} = \frac{1}{1 + E\left(I\left(\hat{\theta}\right)^{-1}\right)}$$

This assumes that $\sigma^2_{E\left(\hat{\theta}|\theta\right)} = \sigma^2_\theta = 1$. Although the formula is often employed in several IRT studies and applications, the underlying assumption may not be true.

Value

Estimated marginal reliability coefficient.

Author(s)

Seewoo Li <cu@yonsei.ac.kr>

References

Green, B.F., Bock, R.D., Humphreys, L.G., Linn, R.L., & Reckase, M.D. (1984). Technical guidelines for assessing computerized adaptive tests. *Journal of Educational Measurement, 21*(4), 347–360. Kim, S. (2012). A note on the reliability coefficients for item response model-based ability estimates. *Psychometrika, 77*(1), 153-162. Kim, S., Feldt, L.S. (2010). The estimation of the IRT reliability coefficient and its lower and upper bounds, with comparisons to CTT reliability statistics. *Asia Pacific Education Review, 11*, 179–188.

Examples

```
## Not run:
Alldata <- DataGeneration(seed = 1,
                          model_D = rep(1, 10),
                          N=500,
                          nitem_D = 10,
                          latent_dist = "2NM",
                          d = 1.664,
                          sd_ratio = 2,
                          prob = 0.3)
```

```
data <- Alldata$data_D

# Analysis

M1 <- IRTest_Dich(data)


# Reliability coefficients
rel <- reliability(M1)

## On the summed-score scale
rel$summed.score.scale$test
rel$summed.score.scale$item

## On the summed-score scale
rel$theta.scale


## End(Not run)
```

`summary.irtest` *Summary of the results*

Description

These functions summarize the outputs (e.g., convergence of the estimation algorithm, parameter estimates, AIC, etc.).

Usage

```
## S3 method for class 'irtest'
summary(object, ...)
```

Arguments

`object`	An `class == "irtest"` object obtained from either `IRTest_Dich`, `IRTest_Poly`, or `IRTest_Mix`.
`...`	Other argument(s) passed on to summarize the results.

Value

A plot of estimated latent distribution.

Index