

# Package ‘penDvine’

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**Type** Package

**Title** Flexible Pair-Copula Estimation in D-Vines using Bivariate Penalized Splines

**Version** 0.2.4

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**Depends** R (>= 2.15.1), lattice, TSP, fda, Matrix, foreach

**Imports** quadprog, latticeExtra, doParallel

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**Description** Flexible Pair-Copula Estimation in D-vines using Bivariate Penalized Splines.

**License** GPL (>= 2)

**LazyLoad** yes

**NeedsCompilation** no

**Repository** CRAN

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## R topics documented:

penDvine-package	2
bernstein	3
cal.Dvine	4
cond.cop	4
Derv1	5
Derv2	6
Dvine	7
f.hat.val	8
marg.likelihood	9
my.IC	10
my.loop	11
new.weights	12
order.Dvine	13

paircopula . . . . .	14
pen.log.like . . . . .	15
pen.matrix . . . . .	16
plot.paircopula . . . . .	17
sim.Dvine . . . . .	18
Winddata . . . . .	19

<b>Index</b>	<b>20</b>
--------------	-----------

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penDvine-package	<i>The package 'penDvine' offers routines for estimating densities and copula distribution of D-vines.</i>
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## Description

The package 'penDvine' offers routines for estimating densities and distribution of D-vines. For details see the description of the function Dvine().

## Details

Package:	penDvine
Type:	Package
Version:	0.2.4
Date:	2015-07-02
License: GPL (>= 2) LazyLoad:	yes

The packages contributes the function 'Dvine()' for estimating densities and distributions of Dvines using penalized splines techniques.

## Author(s)

Christian Schellhase <cschellhase@wiwi.uni-bielefeld.de>

## References

Flexible Pair-Copula Estimation in D-vines using Bivariate Penalized Splines, Kauermann G. and Schellhase C. (2014), Statistics and Computing 24(6): 1081-1100).

## Examples

```
#This examples describes the estimation of a D-vine to winddata,
#available in this package. After the margins are prepared, we estimate a
#D-vine using B-splines with 9 (K+1) marginal knots and penalizing second (m=2)
#order differences (pen=1) of the basis coefficients.

data(Winddata)
ex.data<-Winddata[c(1:100),c(1:4)] #exemplary subdata for fast calculation
```

```
wind.example<-Dvine(ex.data,K=7,pen=1,lambda=100,order.Dvine=FALSE,base="B-spline",m=2,cores=2)
```

---

bernstein

*Flexible Pair-Copula Estimation in D-vines with Penalized Splines*

---

### Description

Calculation of Bernstein Polynomials, following the formula  $\binom{n}{v} x^v (1-x)^{n-v}$  \* (n + 1) .

### Usage

```
bernstein(v,x,n)
```

### Arguments

v	n choose v.
x	Argument between 0 and 1.
n	n choose v.

### Value

Returning a matrix, containing Bernstein Polynomials.

### Note

bernstein and bernstein2 are identical, only the first argument of the functions differs for different applications of 'apply'.

### Author(s)

Christian Schellhase <cschellhase@wiwi.uni-bielefeld.de>

### References

Flexible Pair-Copula Estimation in D-vines using Bivariate Penalized Splines, Kauermann G. and Schellhase C. (2014+), Statistics and Computing (to appear).

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`cal.Dvine`*Flexible Pair-Copula Estimation in D-vines with Penalized Splines*

---

**Description**

Calculating the density of the estimated Dvine at the point(s) `val`.

**Usage**

```
cal.Dvine(obj, val)
```

**Arguments**

`obj` object of class 'penDvine', result of 'Dvine'.  
`val` Values in which the current Dvine should be evaluated.

**Details**

The current Dvine is evaluated in `val` and the corresponding density values are returned.

**Value**

The returning values are the density of the current Dvine at the point(s) '`val`'.

**Author(s)**

Christian Schellhase <cschellhase@wiwi.uni-bielefeld.de>

**References**

Flexible Pair-Copula Estimation in D-vines using Bivariate Penalized Splines, Kauermann G. and Schellhase C. (2014+), Statistics and Computing (to appear).

---

`cond.cop`*Flexible Pair-Copula Estimation in D-vines with Penalized Splines*

---

**Description**

Calculation of the conditional paircopulas.

**Usage**

```
cond.cop(data, coef, K, diff="u2", Index.basis.D, base, q=2)
```

**Arguments**

data	Considered bivariate data, later used as "u1" and "u2".
coef	Considered coefficients of the splines.
K	Number of splines.
diff	Default="u2", alternatively diff="u1". Determining in which direction the expression is differentiated.
Index.basis.D	Vector of indices built in the program before.
base	"B-spline" or "Bernstein".
q	Order of the B-spline, default q=2

**Author(s)**

Christian Schellhase <cschellhase@wiwi.uni-bielefeld.de>

**References**

Flexible Pair-Copula Estimation in D-vines using Bivariate Penalized Splines, Kauermann G. and Schellhase C. (2014+), Statistics and Computing (to appear).

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Derv1	<i>Calculating the first derivative of the paircopula likelihood function w.r.t. parameter b</i>
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**Description**

Calculating the first derivative of the paircopula likelihood function w.r.t. parameter v.

**Usage**

```
Derv1(penden.env, temp=FALSE, lambda=NULL)
```

**Arguments**

penden.env	Containing all information, environment of paircopula().
temp	Default=FALSE, if TRUE temporary calculations of optimal parameters are done.
lambda	Default=NULL, i.e. the saved smoothing parameter lambda in the environment is used. Alternatively, temporary values of lambda are used for optimization of lambda.

**Details**

The calculation of the first derivative of the paircopula likelihood function w.r.t.  $b$  equals

$$s(v, \lambda) = \partial l(v, \lambda) / \partial v = \sum_{i=1}^n \Phi(u_i) / c(u_i, v) - P(\lambda)v$$

with

$$P(\lambda)$$

is the penalty matrix, saved in the environment.

**Value**

Derv1.pen first order derivation of the penalized likelihood function w.r.t. parameter  $v$ .

Derv1.pen is saved in the environment.

**Author(s)**

Christian Schellhase <cschellhase@wiwi.uni-bielefeld.de>

**References**

Flexible Pair-Copula Estimation in D-vines using Bivariate Penalized Splines, Kauermann G. and Schellhase C. (2014+), Statistics and Computing (to appear).

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Derv2	<i>Calculating the second order derivative of the paircopula likelihood function w.r.t. parameter <math>b</math></i>
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---

**Description**

Calculating the second order derivative of the paircopula likelihood function w.r.t. parameter  $v$ .

**Usage**

```
Derv2(penden.env, temp=FALSE, lambda=NULL)
```

**Arguments**

penden.env	Containing all information, environment of paircopula().
temp	Default=FALSE, if TRUE temporary calculations of optimal parameters are done.
lambda	Default=NULL, i.e. the saved smoothing parameter lambda in the environment is used. Alternatively, temporary values of lambda are used for optimization of lambda.

**Details**

We approximate the second order derivative in this approach with the negative fisher information.



**Details**

The calculation of the Dvine is done stepwise. If the option 'order.Dvine' is selected, the order of the first level of the Dvine is specified. From the second level, each paircopula is calculated (parallel or not) until the highest level. The specifications in 'Dvine' are done for every paircopula in the Dvine. There is no option to change parameters for some paircopulas.

**Value**

Returning a list containing

Dvine	an object of class 'Dvine'
log.like	the estimated log-likelihood
AIC	AIC value
cAIC	corrected AIC value
K	Number of K
order	the used order of the first level
S	Sequence seq(1:(dim(data)[2]))
N	Number of observations, that is dim(data)[1]
base	Used basis function

**Author(s)**

Christian Schellhase <cschellhase@wiwi.uni-bielefeld.de>

**References**

Flexible Pair-Copula Estimation in D-vines using Bivariate Penalized Splines, Kauermann G. and Schellhase C. (2014+), Statistics and Computing (to appear).

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<code>f.hat.val</code>	<i>Calculating the actual fitted values 'f.hat.val' of the estimated density function</i>
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**Description**

Calculating the actual fitted values of the response, depending on the actual parameter set  $v$

**Usage**

```
f.hat.val(penden.env, cal=FALSE, temp=FALSE)
```



**Arguments**

penden.env	Containing all information, environment of paircopula()
cal	if TRUE, the final weights of one iteration are used for the calculation of the fitted values.
temp	if TRUE, the iteration for optimal weights is still in progress and the temporary weights are used for calculation of the fitted values.

**Details**

Calculating the actual fitted values of the response, depending on the actual parameter set  $v$ . Multiplying the actual set of parameters  $v$  with the base 'base.den' delivers the fitted values.

**Value**

f.hat.val Fitted values for the current coefficients  
 .f.hat.val is saved in the environment.

**Author(s)**

Christian Schellhase <cschellhase@wiwi.uni-bielefeld.de>

**References**

Flexible Pair-Copula Estimation in D-vines using Bivariate Penalized Splines, Kauermann G. and Schellhase C. (2014+), Statistics and Computing (to appear).

---

marg.likelihood      *Calculating the marginal likelihood*

---

**Description**

Calculating the marginal likelihood of paircopula().

**Usage**

```
marg.likelihood(penden.env, pen.likelihood, temp=FALSE)
```

**Arguments**

penden.env	Containing all information, environment of paircopula().
pen.likelihood	Actual penalized likelihood for calculation, temporary or not.
temp	Default=FALSE, indicating if temporary values throughout iteration are calculated.

**Value**

marg.log.like Marginal log-likelihood, saved in the environment

**Author(s)**

Christian Schellhase <cschellhase@wiwi.uni-bielefeld.de>

**References**

Flexible Pair-Copula Estimation in D-vines using Bivariate Penalized Splines, Kauermann G. and Schellhase C. (2014+), Statistics and Computing (to appear).

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my.IC

*Calculating the AIC-, cAIC- and BIC-value*

---

**Description**

Calculating the AIC-, cAIC- and BIC- value of the paircopula density estimation. Therefore, we add the unpenalized log likelihood of the estimation and the degree of freedom.

**Usage**

```
my.IC(penden.env, temp=FALSE)
```

**Arguments**

penden.env	Containing all information, environment of paircopula()
temp	Default=FALSE, if TRUE temporary values of AIC, cAIC and BIC are calculated.

**Details**

AIC is calculated as  $AIC(\lambda) = -2 * l(\mathbf{u}, \hat{\mathbf{v}}) + 2 * df(\lambda)$

cAIC is calculated as  $AIC(\lambda) = -2 * l(\mathbf{u}, \hat{\mathbf{v}}) + 2 * df(\lambda) + (2 * df * (df + 1)) / (n - df - 1)$

BIC is calculated as  $BIC(\lambda) = 2 * l(\mathbf{u}, \hat{\mathbf{v}}) + 2 * df(\lambda) * \log(n)$

**Value**

AIC	sum of twice the negative non-penalized log likelihood and mytrace
cAIC	corrected AIC.
trace	calculated mytrace as the sum of the diagonal matrix df, which results as the product of the inverse of the penalized second order derivative of the log likelihood with the non-penalized second order derivative of the log likelihood
BIC	sum of twice the non-penalized log likelihood and log(n)

All values are saved in the environment.

**Author(s)**

Christian Schellhase <cschellhase@wiwi.uni-bielefeld.de>

## References

Flexible Pair-Copula Estimation in D-vines using Bivariate Penalized Splines, Kauermann G. and Schellhase C. (2014+), Statistics and Computing (to appear).

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my.loop	<i>Iterative loop for calculating the optimal coefficients 'v'.</i>
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## Description

Calculating the optimal coefficients 'v' iteratively, using quadratic programming.

## Usage

```
my.loop(penden.env)
```

## Arguments

penden.env      Containing all information, environment of pencopula()

## Details

'my.loop' optimates the log-likelihood iteratively. Therefore, the routine checks a) the relative change in the optimal lambda and stops the iteration, if the relative change of lambda is less than one percent. During the calculations of new weights 'v' in the routine 'new.weights', most of the values are called '.temp'. This add on underlines the temporarily values. Alternatively b) for fixed lambda, 'my.loop' checks the relative change in the weights. If the change of a) the optimal lambda or b) of the basis coefficients 'v' are greater than one percent, the the real values are overwritten with the '.temp' values.

## Value

liste            The results of each iteration are written in a matrix called 'liste', saved in the environment. 'liste' contains the penalized log-likelihood, the log-likelihood, 'lambda' and the weights 'v'.

## Author(s)

Christian Schellhase <cschellhase@wiwi.uni-bielefeld.de>

## References

Flexible Pair-Copula Estimation in D-vines using Bivariate Penalized Splines, Kauermann G. and Schellhase C. (2014+), Statistics and Computing (to appear).

---

new.weights                      *Calculating new weights v.*

---

### Description

Calculating new weights  $v$  using quadratic programming.

### Usage

```
new.weights(penden.env, lambda.temp=NULL)
```

### Arguments

penden.env	Containing all information, environment of paircopula()
lambda.temp	Default=NULL, if optimal lambda is calculated, the lambda are saved temporarily, also the resulted coefficients are saved temporarily until some convergences.

### Details

The new weights are calculated solving a quadratic program. Therefore, the derivatives of first and second order are needed, 'Derv1.pen' and 'Derv2.pen'. Moreover, we have to fulfill the side conditions  $v \geq 0$ ,  $\sum(v)=1$  and that the marginal densities are uniform. All side conditions are saved as 'AA.help' in the environment.

If the quadratic program does not find a new feasible solution, the whole program terminates. For solving the quadratic program, we use the function 'solve.QP' from the R-package 'quadprog'.

### Value

ck.val.temp	Calculated new values for the weights 'v'. The add on 'temp' means, that there is a check in the next step if the weights 'v' have been converted (in the case of fixed lambda). If converted, the new values 'ck.val.temp' are unnoted. If not converted, 'ck.val.temp' become the ordinary 'ck.val' for the next iteration. This check is done in my.loop. If the optimal value of lambda is calculated, the coefficients 'ck.val.temp' become the ordinary 'ck.val' for the next iteration if lambda is converted. t
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'ck.val.temp' is saved in the environment.

### Author(s)

Christian Schellhase <cschellhase@wiwi.uni-bielefeld.de>

### References

Flexible Pair-Copula Estimation in D-vines using Bivariate Penalized Splines, Kauermann G. and Schellhase C. (2014+), Statistics and Computing (to appear).

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order.Dvine	<i>Ordering the first level of the Dvine.</i>
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**Description**

Ordering the first level of the Dvine, depending on the pairwise cAIC-values.

**Usage**

```
order.Dvine(help.env)
```

**Arguments**

help.env            Containing all information, environment of Dvine()

**Details**

Beginning in the top tree level of a p-dimensional D-vine, we calculate all  $\binom{p}{2}$  'p over 2' marginal pairwise copulas fitted by penalized splines. For each pair (i,j) this gives the fitted maximized likelihood value. We order the variable pairs, subject to their increasing estimated pairwise cAIC and start with the pair of covariates with lowest estimated cAIC. We now select the pairs of variables such that the resulting selection gives a tree. The problem of finding this selection is equivalent to solve a traveler salesman problem by interpreting the cAIC as distance measure between two variables. Once this problem is solved, the specification of the first tree level completely defines the D-vine

**Value**

order                Calculated order of the first level of the Dvine, saved in the environment.

**Author(s)**

Christian Schellhase <cschellhase@wiwi.uni-bielefeld.de>

**References**

Flexible Pair-Copula Estimation in D-vines using Bivariate Penalized Splines, Kauermann G. and Schellhase C. (2014+), Statistics and Computing (to appear).

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paircopula	<i>Flexible Pair-Copula Estimation in D-vines using Bivariate Penalized Splines</i>
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### Description

Calculating paircopula with penalized B-splines or penalized Bernstein polynomials

### Usage

```
paircopula(data,K=8,base="Bernstein",max.iter=30,lambda=100,
           data.frame=parent.frame(),m=2,fix.lambda=FALSE,pen=1,q=2)
```

### Arguments

data	'data' contains the data. 'data' has to be a matrix or a data.frame with two columns.
K	K is the degree of the Bernstein polynomials. In the case of linear B-spline basis functions, K+1 nodes are used for the basis functions.
base	Type of basis function, default is "Bernstein". An alternative is base="B-spline".
max.iter	maximum number of iteration, the default is max.iter=30.
lambda	Starting value for lambda, default is lambda=100.
data.frame	reference to the data. Default reference is the parent.frame().
m	Indicating the order of differences to be penalised. Default is "m=2".
fix.lambda	Determining if lambda is fixed or if the iteration for an optimal lambda is used, default 'fix.lambda=FALSE'.
pen	'pen' indicates the used penalty. 'pen=1' for the difference penalty of m-th order. 'pen=2' is only implemented for Bernstein polynomials, it is the penalty based on the integrated squared second order derivatives of the Bernstein polynomials. Due to numerical difficulties handling the integral of Bernstein polynomials (that is the beta function), this approach works only for $K \leq 15$ .
q	Order of B-spline basis, i.e. default q=2 means linear B-spline basis.

### Details

Each paircopula is calculated using Bernstein polynomials or B-spline densities as basis functions. Optimal coefficients and optimal penalty parameter lambda are selected iteratively using quadratic programming.

### Value

Returning an object of class 'paircopula', consisting of the environment 'penden.env', which includes all values.

**Author(s)**

Christian Schellhase <cschellhase@wiwi.uni-bielefeld.de>

**References**

Flexible Pair-Copula Estimation in D-vines using Bivariate Penalized Splines, Kauermann G. and Schellhase C. (2014+), Statistics and Computing (to appear).

---

pen.log.like	<i>Calculating the log likelihood</i>
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**Description**

Calculating the considered log likelihood.

**Usage**

```
pen.log.like(penden.env, cal=FALSE, temp=FALSE)
```

**Arguments**

penden.env	Containing all information, environment of paircopula()
cal	if TRUE, the final weights of one iteration are used for the calculation of the penalized log likelihood.
temp	if TRUE, the iteration for optimal weights is still in progress and the temporary weights are used for calculation.

**Details**

The calculation depends on the estimated weights  $v$ , the penalized splines  $\Phi$  and the penalty paramters  $\lambda$ .

$$l(v, \lambda) = \sum_{i=1}^n \left[ \log \left\{ \sum_{i=1}^n \Phi(u_i) \right\} v \right] - \frac{1}{2} v^T P(\lambda) b$$

The needed values are saved in the environment.

**Value**

pen.log.like	Penalized log likelihood of the paircopula density.
log.like	Log-Likelihood of the paircopula density.

The values are saved in the environment.

**Author(s)**

Christian Schellhase <cschellhase@wiwi.uni-bielefeld.de>

**References**

Flexible Pair-Copula Estimation in D-vines using Bivariate Penalized Splines, Kauermann G. and Schellhase C. (2014+), Statistics and Computing (to appear).

---

pen.matrix	<i>Calculating the penalty matrix P</i>
------------	---

---

**Description**

Calculating the penalty matrix P depends on the used basis function and the selected kind of penalty.

**Usage**

```
pen.matrix(penden.env)
```

**Arguments**

penden.env      Containing all information, environment of paircopula().

**Details**

If paircopula is selected with 'pen=1', the differences of order 'm' are penalized. Only for Bernstein polynomials: If paircopula is selected with 'pen=2', the integrated squared second order derivatives are used as penalty.

**Value**

DDD.sum          Penalty matrix P

Matrix is saved in the environment.

**Author(s)**

Christian Schellhase <cschellhase@wiwi.uni-bielefeld.de>

**References**

Flexible Pair-Copula Estimation in D-vines using Bivariate Penalized Splines, Kauermann G. and Schellhase C. (2014+), Statistics and Computing (to appear).



---

plot.paircopula      *Flexible Pair-Copula Estimation in D-vines with Penalized Splines*

---

## Description

Plotting a paircopula of class 'paircopula'.

## Usage

```
## S3 method for class 'paircopula'
plot(x, val=NULL, marg=TRUE, plot.view=TRUE, int=FALSE, main.txt=NULL,
      sub.txt=NULL, contour=FALSE, cuts=20, cex=1, cex.axes=1,
      xlab=NULL, ylab=NULL, zlab=NULL, xlim=NULL, ylim=NULL, zlim=NULL,
      show.observe=FALSE, margin.normal=FALSE, ...)
```

## Arguments

x	object of class 'paircopula', result of function 'paircopula'.
val	Default val = NULL, one can calculate the estimated density/distribution for bivariate vector, e.g. val=c(0.5,1).
marg	Default = TRUE, plotting the marginal densities.
plot.view	Default = TRUE, if 'FALSE' no plot is shown, e.g. for calculations with val != NULL.
int	Default = FALSE, if TRUE, the integral, i.e. the distribution of the copula density is plotted.
main.txt	Default = NULL shows 'K' and the value of lambda.
sub.txt	Default = NULL shows the log-likelihood, the penalized log-likelihood and the cAIC-value of the estimation.
contour	If TRUE, a contour plot is shown. Default = FALSE.
cuts	Number of cuts for the contour plots, if contour=TRUE.
cex	Default = 1, determining the size of the main of the plot.
cex.axes	Default = 1, determining the size of the labels at the axes.
xlab	Default = NULL and no text is printed at the xlab
ylab	Default = NULL and no text is printed at the ylab
zlab	Default = NULL and 'density' is printed at the zlab for int=FALSE and 'distribution' for int=TRUE.
xlim	Default = NULL, changes the range for the values of x in the case of a contour plot.
ylim	Default = NULL, changes the range for the values of y in the case of a contour plot.
zlim	Default = NULL and the range for the values of z are the range of calculated values.

show.observ      Default = FALSE. If TRUE, plotting the original observation into a contourplot.  
margin.normal    Default = FALSE. If TRUE, the plot is presented with margins following standard normal distribution.  
...                further arguments

**Value**

If 'val' is not NULL, the function returns a matrix with the calculated density or distribution values for the set 'val'.

**Author(s)**

Christian Schellhase <cschellhase@wiwi.uni-bielefeld.de>

**References**

Flexible Pair-Copula Estimation in D-vines using Bivariate Penalized Splines, Kauermann G. and Schellhase C. (2014+), Statistics and Computing (to appear).

---

sim.Dvine

*Flexible Pair-Copula Estimation in D-vines with Penalized Splines*

---

**Description**

Simulating a p-dimensional vector of the fitted p-dimensional D-vine.

**Usage**

```
sim.Dvine(DV, N=NULL)
```

**Arguments**

DV                object of class 'penDvine', result of 'Dvine'.  
N                 Number of simulated p-dimensional pairs from the fitted D-vine 'DV'.

**Value**

The returning values are simulated values of the current Dvine.

**Author(s)**

Christian Schellhase <cschellhase@wiwi.uni-bielefeld.de>

**References**

Flexible Pair-Copula Estimation in D-vines using Bivariate Penalized Splines, Kauermann G. and Schellhase C. (2014+), Statistics and Computing (to appear).

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Winddata	<i>Maximal daily windspeed in khm.</i>
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---

**Description**

Maximal daily windspeed in khm at Frankfurt, Berlin, Bremen, Munich and Cuxhaven, observed in the time from 01/01/2000 until 31/12/2011.

**Usage**

```
data(Winddata)
```

**Format**

A data frame with 507 observations of the following 2 variables.

Date Date

Frankfurt Observations in Frankfurt

Berlin Observations in Berlin

Bremen Observations in Bremen

Munich Observations in Munich

Cuxhaven Observations in Cuxhaven

**Note**

The data is available at the internet page of the 'Deutsche Wetterdienst' DWD, [www.dwd.de](http://www.dwd.de). 'Winddata-original' contains the original data, 'Winddata' contains marginal fitted t-distributions of the data.

# Index

- \* **datasets**
  - Winddata, [19](#)
- \* **math**
  - Derv1, [5](#)
  - Derv2, [6](#)
  - my.IC, [10](#)
  - my.loop, [11](#)
- \* **nonparametric**
  - f.hat.val, [8](#)
  - marg.likelihood, [9](#)
  - new.weights, [12](#)
  - order.Dvine, [13](#)
  - pen.log.like, [15](#)
  - pen.matrix, [16](#)

bernstein, [3](#)  
bernstein2(bernstein), [3](#)

cal.Dvine, [4](#)  
cond.cop, [4](#)

Derv1, [5](#)  
Derv2, [6](#)  
Dvine, [7](#)

f.hat.val, [8](#)

marg.likelihood, [9](#)  
my.IC, [10](#)  
my.loop, [11](#)

new.weights, [12](#)

order.Dvine, [13](#)

paircopula, [14](#)  
pen.log.like, [15](#)  
pen.matrix, [16](#)  
penDvine-package, [2](#)  
plot.paircopula, [17](#)

sim.Dvine, [18](#)

Winddata, [19](#)  
Winddata-original (Winddata), [19](#)